

REGGE DESCRIPTION OF $\pi\pi$, πK , πN , KN AND NN FORWARD SCATTERING ABOVE $E_{kin} > 1 \text{ GeV}$

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We review our recent description of high energy forward hadronic cross sections and in particular of pion scattering data by means of Regge theory. We obtain a prediction for the total pp cross section at LHC. In addition, and contrary to some suggestions in the literature we show that standard Regge behavior can accommodate both crossing symmetry and factorization. Some consequences for low energy $\pi\pi$ scattering are briefly discussed.

1 Introduction

A correct description of $\pi\pi$ scattering at high energies is crucial to achieve a very precise dispersive evaluation of $\pi\pi$ low energy observables. This is of particular interest to test Chiral Perturbation Theory (ChPT), and in particular the values of the chiral parameters and of the chiral condensate. Since $\pi\pi$ scattering amplitudes are very constrained by analyticity, dispersive approaches, using data in a larger energy range, can improve the information from low energy data alone, which are not very reliable and are affected by large systematics.

At high energies, the formalism able to deal with hadronic cross sections is Regge theory, which is as much part of QCD as ChPT. It describes amplitudes in terms of *Regge poles*, which are complicated objects related to the t channel exchange of resonances, like the reggeized ρ in the isospin 1 channel, or the *Pomeron* when no quantum numbers are exchanged. All Regge pole contributions decrease at large s , except that of the Pomeron, so that all $\pi\pi$ total cross sections, tend to a common value at sufficiently high energies ($\simeq 20 \text{ GeV}$), denoted σ^∞ , (Pomeranchuk Theorem). In addition, Regge theory, relates different processes thanks to *factorization*, i.e.:

$$\text{Im } F_{A+B \rightarrow A+B}(s, t) \simeq f_A(t) f_B(t) (s/\hat{s})^{\alpha_R(t)}, \quad \hat{s} = (1 \text{ GeV})^2. \quad (1)$$

Let us recall that total cross sections are related to forward scattering amplitudes by: $\sigma_{AB} = 4\pi^2 \text{Im } F_{A+B \rightarrow A+B}(s, 0) / \lambda^{1/2}(s, m_A^2, m_B^2)$, with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The $(s/\hat{s})^{\alpha_R(t)}$ behavior depends on the Regge pole R , whereas the $f_i(t)$ factors depend on the particles in the initial state. It is then possible to obtain $\pi\pi$ Regge amplitudes from those of πN and NN . The (s, t) applicability range of the Regge formalism will be discussed below, but QCD specifies that $s \gg \Lambda_{QCD}^2$ and $s \gg |t|$.

In the early seventies, when only phase shift analysis up to 2 GeV ¹ were available for $\pi\pi$ scattering, and QCD was not fully developed, it was found that certain crossing symmetry sum

rules were not satisfied together with factorization², which, at that time, implied $\sigma^\infty \simeq 15$ mb from πN and NN data. (In practice, ∞ means ~ 20 GeV, since above that energy hadronic cross sections grow, as we will see below). Thus, it was suggested that factorization could be “badly broken”, and that $\sigma^\infty = 6 \pm 5$ mb. Still, the same authors³ remarked later that σ^∞ should be raised at least to 8.3 mb, and that recent high energy measurements⁴ of total $\pi\pi$ cross sections found $\sigma_{\pi^+\pi^-}^{tot} = 13.5 \pm 2.5$ mb at 32 GeV^2 . In the late seventies other authors⁵ used the correct value in their analysis, but when several other experiments in the late seventies confirmed this result, the interest in Regge theory and pion scattering had already faded away.

In recent years, there is a renewed interest due to the implications for Chiral Perturbation Theory, through the precise determination of the meson-meson scattering lengths, which requires the use of dispersive integrals. Unaware of the last experiments⁴, all these studies^{6,7,8} have used a reanalysis⁶ of the $\pi\pi$ Regge description with $\sigma^\infty = 5 \pm 3$ mb.

2 Regge description of hadronic total cross sections including $\pi\pi$ scattering

For the previous reasons we have recently presented⁹ a Regge fit able to describe NN , $\pi^\pm N$, $K^\pm N$ and $\pi\pi$ from $E_{kin} \simeq 1 \text{ GeV}$ up to about $\simeq 16 \text{ GeV}$. We refer the readers to⁹ for details. Note that this new fit using $\pi\pi$ data is compatible, but more precise than a similar parametrization¹⁰, obtained from factorization before we rediscovered the existing high energy data. In Fig.1a, we show our fit to $\pi^\pm N$ and $(pp + p\bar{p})/2$ and $(K^+p + K^-p)/2$ cross sections, (these combinations basically only depend on the Pomeron and the ρ). Here we use a very simple parametrization of the Pomeron with $\alpha_P(0) = 1$, although it is well known that hadronic cross sections grow at large s . Nevertheless, as shown in Fig.1b, with a slight modification of the Pomeron channel, to include a soft logarithm, the hadronic forward cross sections are described up to the multi-TeV region. We predict, for instance, the total pp cross section at LHC to be $\sigma_{pp}^{tot}(14) \text{ TeV} = 108 \pm 4 \pm 4$ mb, in remarkable agreement with the most recent Regge analysis¹¹: $111.5 \pm 1.2^{+4.1}_{-2.1}$. Note also that, at low energies, Fig.1b overlaps with Fig.1a.

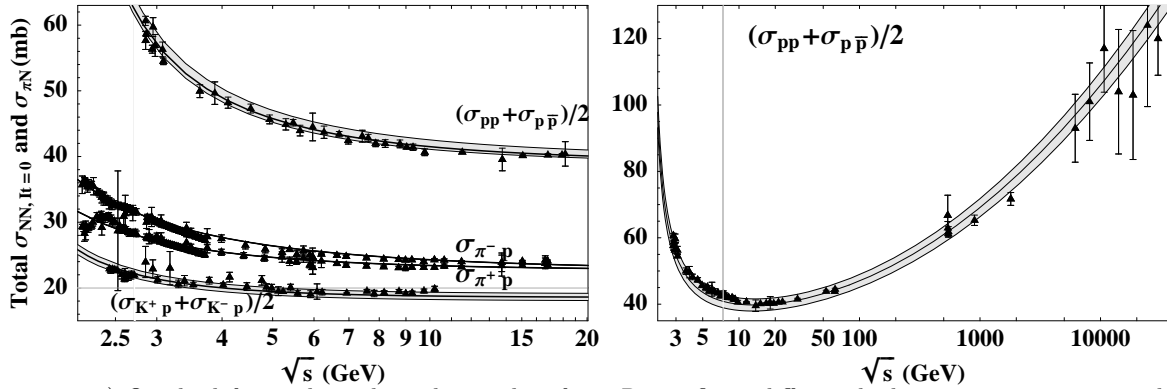


Figure 1: a) On the left panel we show the results of our Regge fit to different hadronic cross sections, with a simple Pomeron parametrization $\alpha_P(0) = 1$, valid up to $E_{kin} \simeq 16 \text{ GeV}$, and assuming factorization. b) Adding a logarithmic term to the Pomeron the description can be easily extended up to the multi TeV range.

However, the raise in hadronic forward cross sections is negligible up to 20 GeV, and is irrelevant for the integrals used in meson-meson dispersive approaches. Hence, it is sufficient to use the simplest Pomeron parametrization of Fig 1a. assuming that factorization is also valid for $\pi\pi$ scattering to obtain the results in Figs.2, represented as solid lines. The thin gray bands around them correspond to our error bars. We find a remarkable agreement with the measured $\pi\pi$ cross sections above 2 GeV. Between 1.4 and 2 GeV different sets of data are in conflict, note however that our result falls between the different sets and that it matches with the points at 1.42 GeV from phase shift analysis (the stars or the dotted lines). Finally, we

find that all $\pi\pi$ amplitudes flatten around 20 GeV and fall within 0.5 mb of an average value $\sigma_{\pi\pi}^{tot}(20\text{GeV}) = 13.4 \pm 0.6 \text{ mb}$. This value is totally dominated by the Pomeron and, assuming factorization, is in very good agreement with the Regge parameterization¹² that will appear in the Review of Particle Properties 2004¹³, $\simeq 12.3 \pm 0.3 \text{ mb}$. For comparison we show, as dashed lines, the results using the parameterizations in⁶ with their uncertainties (light gray bands).

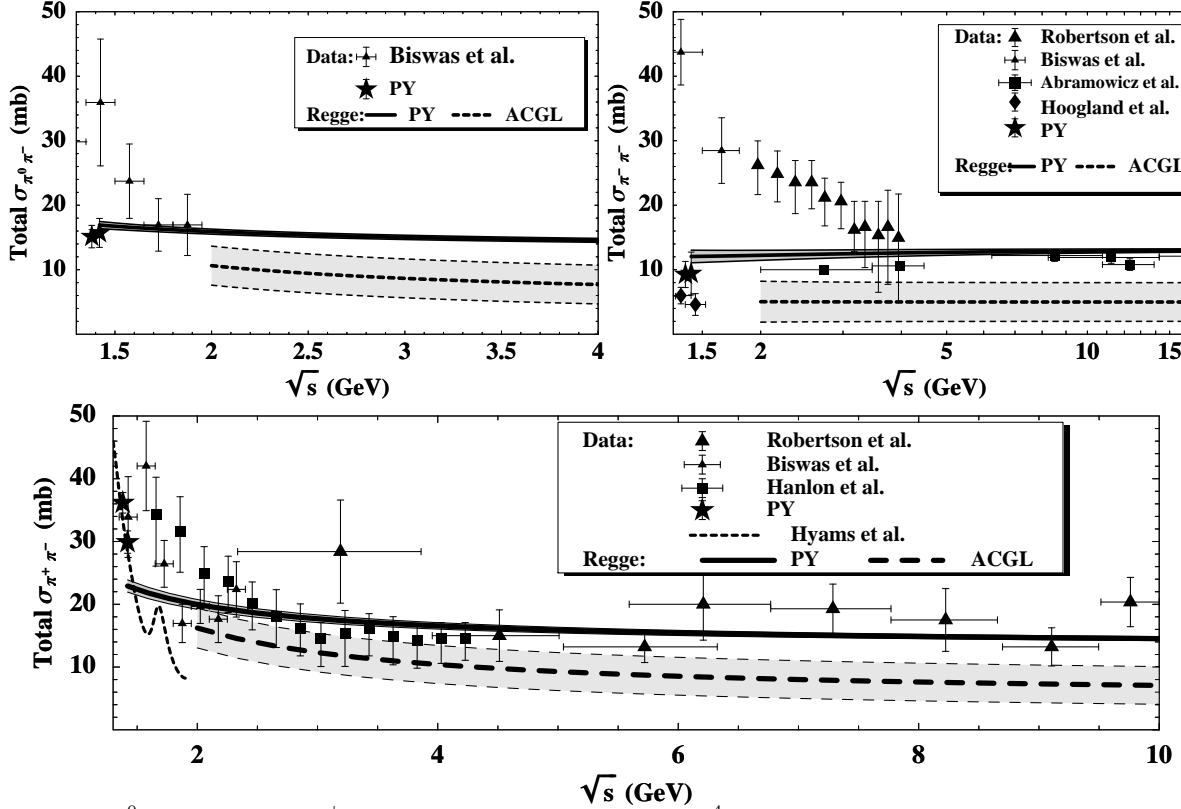


Figure 2: $\pi^0\pi^-$, $\pi^-\pi^-$ and $\pi^+\pi^-$ total cross sections and data⁴. The solid lines correspond to our Regge fit assuming factorization. The dashed correspond lines to the parametrization⁶ most frequently used in recent dispersive studies of $\pi\pi$ scattering. The gray bands cover the uncertainties of each parametrization. The PY points have been obtained¹⁰ from parameterizations of phase shift analysis.

3 $\pi\pi$ scattering and crossing sum rules

We have also checked that our Regge description of $\pi\pi$ is consistent with crossing sum rules. In particular, our parametrization without fitting⁹, satisfies the following two crossing sum rules:

$$\int_{4M_\pi^2}^{\infty} ds \left\{ \frac{4\text{Im}F'^{I_s=0}(s,0) - 10\text{Im}F'^{I_s=2}(s,0)}{s^2(s-4M_\pi^2)^2} - 6(3s-4m_\pi^2) \frac{\text{Im}F'^{I_s=1}(s,0) - \text{Im}F^{I_s=}(s,0)}{s^2(s-4M_\pi^2)^3} \right\} = 0,$$

$$\int_{M_\pi^2}^{\infty} ds \frac{\text{Im}F^{(I_t=1)}(s,4M_\pi^2) - \text{Im}F^{(I_t=1)}(s,0)}{s^2} - \int_{M_\pi^2}^{\infty} ds \frac{8M_\pi^2[s-2M_\pi^2]}{s^2(s-4M_\pi^2)^2} \text{Im}F^{(I_s=1)}(s,0) = 0.$$

where F' means $\partial F / \partial \cos\theta$, θ being the scattering angle.

For these calculations we need the t dependence of the Regge expressions, obtained from fits to the slopes of differential cross sections (see⁹ for detailed expressions). These fits are not unique in the literature, but fortunately they agree numerically for small values of t . For the low t we are interested in, $\sqrt{|t|} = 0.28 \text{ GeV}$, our t behavior lies on the safe side¹⁴.

Both sum rules are well satisfied⁹ with the recent and most precise Regge parameters obtained from the fits in the previous section. The first one is dominated by the Pomeron, and

can be used to constrain the poorly known $I = 2$ Regge exchange. The second one is dominated at high energy by the ρ and at low energy by the P wave, the best known. In this way one can fix with even greater precision the Regge ρ parameters, used to obtain Fig.2.

The old mismatches in these sum rules were due to the use of the CERN-Munich phase shift analysis¹ from $\sqrt{s} = 1.42$ GeV up to 2 GeV. They have been represented by a dotted line in Fig.2, where we can see that they are incompatible with measurements of total cross sections. These data are also in conflict with many other considerations^{9,16}.

4 Relevance for $\pi\pi$ scattering at low energies

In a recent paper, we pointed out that the $\pi\pi$ parameterizations given in⁷, do not satisfy the Olsson sum rule by 2.5 standard deviations and the a_{00} and a_{0+} Froissart Gribov dispersive representations by about 4 to 5 standard deviations. Other dispersive analysis⁸ do not have these conflicts because their error bars are about a factor of three larger than those in⁷. In¹⁵ it was argued that this was due to a faulty Regge representation that violated crossing. As shown here, our recent results⁹ show that this is not the case, since our Regge formalism describes data and satisfies crossing sum rules. The discrepancy remains.

Acknowledgments

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